Superconducting Qubits Coupled to Torsional Resonators

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We propose a scheme of strong and tunable coupling between a superconducting phase qubit and nanomechanical torsional resonator. In our scheme the torsional resonator directly modulates the largest energy scale (the Josephson coupling energy) of the phase qubit, and the coupling strength is very large. We analyze the quantum correlation effects in the torsional resonator as a result of the strong coupling to the phase qubit.

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I. INTRODUCTION

Probing quantum mechanical properties of macroscopic objects is believed to be a key to understand the border between the classical and quantum physics. Nanomechanical resonators, which have high frequency of gigahertz and low dissipation, provide a tangible system to study such macroscopic quantum phenomena[1–3]. Coupling the resonator to the superconducting qubits has attracted great theoretical interest as it provides a way of control and detect the quantum behavior of the resonator[4–12] and a prototypical experiment has recently been demonstrated.[13, 14]

Besides the fundamental aspect of the system, a nanomechanical resonator prepared in a squeezed state can improve its noise properties, upon which the limit of force detection sensitivity is based, beyond the standard quantum limit.[15] An architecture for a scalable quantum computation has also been suggested based on the integration of the nanomechanical resonators with the superconducting phase qubits.[16, 17]

In this paper, we propose a scheme of strong and tunable coupling between a superconducting phase qubit and nanomechanical torsional resonator. In our scheme, the direct modulation of the largest energy scale of the phase qubit enables a large coupling strength. This distinguishes our scheme from other previously proposed schemes. For example, in Ref. [18], the flexural vibrational modes were coupled to a charge qubit by modulating the Josephson energy, which in their case is one of the smallest energy scale of the qubit system. We analyze the quantum correlation effects in the torsional resonator and also provide the noise analysis, which shows that our scheme is feasible experimentally at the level of present technology.

The rest of the paper is organized as follows: In Section II we summarize the basic operational mechanism of the phase qubit and the characteristics of the torsional resonator. In Section III we analyze the coupling mechanism between the phase qubit and the torsional resonator. The reduced coupling constant is expressed in terms of the control parameters of the phase qubit and the torsional resonator. In Section IV we discuss the possible quantum correlations effects, especially, the squeezing of the torsional vibration mode, in the strong coupling limit. In Section V we provide a detailed noise analyses in a possible experimental realization of the scheme. Finally the paper is concluded in Section VI.

II. QUBIT AND RESONATOR

A superconducting phase qubit consists of a double Josephson junction (Fig. 1) of small size, and is described by the Hamiltonian of the form

$$H_{\rm qubit} = E_C n^2 - 2E_J\cos(\pi f)\cos(\varphi - \pi f)\,, \qquad (1)$$

where the number n of Cooper pairs that has tunneled through the double junction and the phase difference φ across the junction are quantum mechanical conjugate variables, i.e., $[\varphi, n] = i$. Here $E_C = (2e)^2/2C \sim 10\,\mathrm{neV}$ is the charging energy of the double junction with total capacitance C, $E_J \sim 50\,\mathrm{meV}$ is the Josephson coupling energy of each junction,[19] f is the external flux (in units of the flux quantum $\Phi_0 = h/2e$) threading the loop. In Eq. (1), the effective Josephson coupling $E_J^{\rm eff} = 2E_J\cos(\pi f)$ of a phase qubit is controlled by the external flux f. A phase qubit typically operates in the range where $k_BT \ll E_C \ll E_J$, and uses as its computation basis the two lowest-energy states $|0\rangle$ and $|1\rangle$ con-

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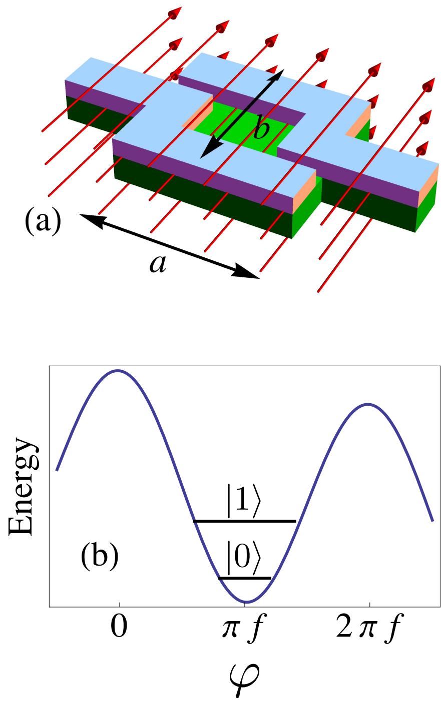

FIG. 1: (color on-line) (a) A schematic of superconducting phase qubit coupled to torsional resonator. The arrows denote the external magnetic field. (b) A schematic of the energy levels corresponding to the logical basis states $|0\rangle$ and $|1\rangle$ of a phase qubit.

fined in the potential well around $\varphi = \pi f$; see Fig. 1(b). Within the subspace spanned by the computational basis, the qubit Hamiltonian (1) can be written as

$$H_{\rm qubit} \approx -\frac{1}{2}\Omega\sigma_z \tag{2}$$

where $\sigma_x, \sigma_y, \sigma_z$ are the Pauli matrices. The level splitting Ω can be estimated by $\Omega \approx \sqrt{2E_C E_J^{\rm eff}} \sim 40\,\mu{\rm eV} \sim 2\pi \times 10\,{\rm GHz}$.[20, 21]

The torsional vibration mode of the substrate is described by a harmonic oscillator Hamiltonian

$$H_{\rm osc} = \frac{P_\theta^2}{2I} + \frac{1}{2}I\omega_0^2\theta^2 \tag{3}$$

where P_θ is the (angular) momentum conjugate to θ, $I \sim 10^{-28}$–$10^{-32}\,{\rm kg{\cdot}m^2}$ is the rotational moment of inertia of the torsional resonator, and $\omega_0/2\pi \sim 8$–$800\,{\rm MHz}$ is the vibration frequency. The fluctuations of the angle θ can be characterized by the parameter $\theta_0 \equiv \sqrt{\hbar/I\omega_0}$, which is the fluctuation in the ground state. In typical experimental situations $\theta_0 \sim 10^{-6}$–$10^{-7}\,{\rm rad}$ depending on the values of I and ω_0.

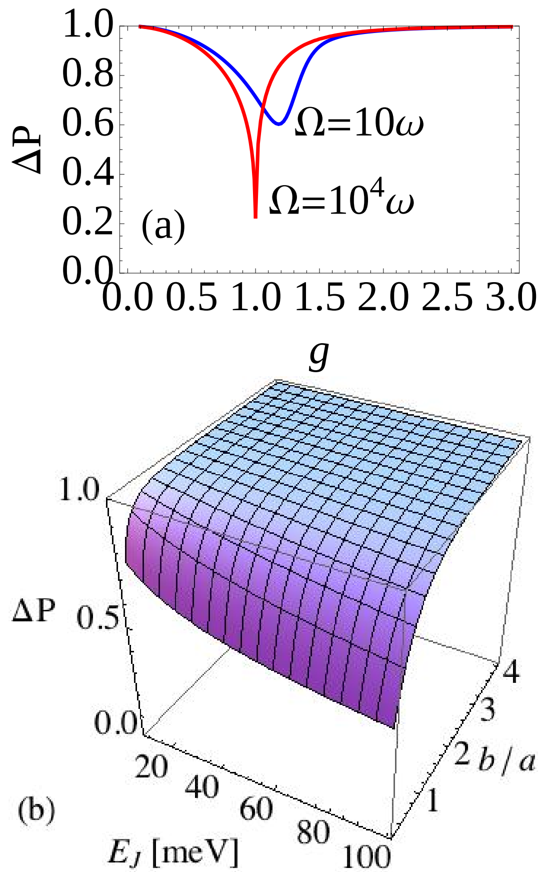

FIG. 2: (a) Squeezing of the torsional resonator as a function of the reduced coupling strength g for $\Omega = 10\omega$ and $\Omega = 10^4\omega$. (b) Squeezing of the torsional resonator as a function of the Josephson energy E_J and the ratio b/a of the lateral sizes of the phase qubit.

III. SPIN-RESONATOR COUPLING

With the qubit put on the torsional resonator as in Fig. 1, the effective flux f in the qubit Hamiltonian (1) is modulated as $f = f_0 \sin\theta$, where θ is measured relative to the direction of the external magnetic field, and hence the qubit is coupled to the torsional vibration mode. We point out a key advantage of this qubit-resonator coupling scheme: As mentioned above, the phase qubit operates in the regime, where E_J is the largest energy scale. The torsional vibration directly modulates this largest energy scale. This means that the coupling between the qubit and the torsional vibrational mode can be large as demonstrated below.

If we apply the external magnetic field parallel to the phase qubit plane, then the flux modulation is given by $f = f_0(\theta - \theta_e)$. Here f_0 is the maximum magnetic flux (i.e., the value when the field is due perpendicular to the qubit plane) and θ_e is the angle at equilibrium measured from the direction of external field. We assume that $\theta_e = 0$ (non-zero θ_e slightly decreases the coupling strength by factor $\sin\theta_e$). Within the two-level approximation, the

total Hamiltonian is given by

$$H = -\frac{1}{2}\Omega\sigma_z + \frac{1}{2}g\sqrt{\Omega\omega}(a + a^\dagger)\sigma_x + \omega a^\dagger a\,. \qquad (4)$$

where g is the dimensionless *reduced coupling constant* between the phase qubit and the torsional resonator. Note that the oscillator frequency has been slightly renormalized from ω_0 to

$$\omega \equiv \omega_0\sqrt{1 + 2(\pi f_0)^2 E_J/I\omega_0^2} \qquad (5)$$

due to the coupling to the phase qubit. The renormalization of the frequency $\omega_0 \to \omega$ also renormalizes the quantum fluctuation angle θ_0 to

$$\theta_1 \equiv \sqrt{\frac{\hbar}{I\omega}}\,. \qquad (6)$$

The coupling constant g in this case is given by

$$g = \pi f_0\sqrt{\frac{2E_J}{I\omega^2}} \qquad (7)$$

The effective Hamiltonian (4) is the well-known cavity-QED (quantum electrodynamics) Hamiltonian for the atom-light interaction in an optical cavity. For optical cavities, the two-level system (or "spin") is at resonance with the oscillator ($\Omega \sim \omega$), and the *coupling energy* $g\sqrt{\Omega\omega}$ is 10^{-6} times smaller, at best, than ω. In such cases, it is customary to make a so-called rotating-wave approximation (RWA), which leads to the Jaynes-Cummings model [22],

$$H \approx -\frac{1}{2}\Omega\sigma_z + \frac{1}{2}g\sqrt{\Omega\omega}(a\sigma_+ + a^\dagger\sigma_-) + \omega a^\dagger a \qquad (8)$$

where $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$. The ground state of the Jaynes-Cumming model (8) does not exhibit any quantum correlation effects of particular interest. As the coupling energy $g\sqrt{\Omega\omega}$ increases ($g \gtrsim 10^{-3}$), however, the RWA breaks down, and the ground states start to exhibit strong quantum correlation effects such as squeezing of the oscillation mode (Fig. 2), as discussed below.

IV. STRONG COUPLING LIMITS

Before we discuss the "strong" coupling limit of the qubit-resonator composite system, we need to distinguish the limit from the *conventional* strong coupling limit. The effective qubit-resonator Hamiltonian (4) is an example of a more general class of spin-boson models, which are commonly achieved in optical cavities. Conventionally, for optical cavities, the strong coupling limit means the coupling constant larger than the energy dissipation rate γ, so that the coherent interaction between the two-level system and the oscillator can be maintained. In our case, we push the limit even further and mainly concern about the regime, where the ground state of the qubit-resonator composite system exhibits non-trivial quantum correlation effects. In this paper, we will use the squeezing in the vibrational mode as the measure of the non-trivial quantum correlation effects.

Despite its simple form of the Hamiltonian (4) the spin-boson model has turned out to be highly non-trivial beyond the Jaynes-Cumming or RWA regime [23]. In particular, the spin-boson model (4) is known to have a strong squeezing effect in its ground state when the coupling energy $g\sqrt{\Omega\omega}$ is comparable to the geometric mean $\sqrt{\Omega\omega}$ of the two energy scales Ω and ω; see Fig. 2 (a). The manipulation of squeezed optical modes using the light-atom interaction in an optical cavity is by now standard [see, e.g., 24]. However, an important difference is that in our scheme the squeezing is achieved in the *static* ground state of the system. To the contrary, in optical cavities it can be created only by *dynamical* procedures due to the weak coupling. That is, the two-level atoms should be prepared in a special quantum state by means of a sequence of optical pulses before they interact with the cavity modes and the atom-cavity interaction time should be tune precisely depending on the initial state of the atoms. The resulting squeezing is thus much more difficult and less stable than in our scheme. Another important difference is that in conventional optical cavity $\Omega \sim \omega$ whereas in our scheme the detuning is very large ($\Omega/\omega \sim 10^4$; recall that the actual coupling energy in natural units is given by $g\sqrt{\Omega\omega}$).

The detailed analysis and discussion on the squeezing effect in the strong-coupled spin-boson model is out of scope of this paper. Here we merely refer the readers to recent discussions in [25, 26], and define the strong coupling (also called ultra-strong coupling) limit by the condition $g \sim 1$.

The coupling constant g in (7) is estimated to be $g \simeq 0.5$, surpassing the strongest coupling strength achieved so far in the previous qubit-oscillator coupling schemes, already in the macroscopic samples of lateral size of several micrometers ($I \sim 10^{-28}\,\mathrm{kg{\cdot}m^2}$) [27–29]. Here we have put $f_0 \simeq 100$ assuming an external field of $B \sim 0.1\,\mathrm{T}$ (the lower critical field of Nb, for example, is as large as $150\,\mathrm{T}$ [30]). One can enhance the coupling strength even further by using the phase qubit with the lateral size of several hundred nanometers and by designing its geometric shape so that the loop containing the double Josephson junction is longer along the axis, namely, $a > b$ in Fig. 1 keeping the loop area $a \times b$ the same (for example, $a \sim 2\,\mu\mathrm{m}$ and $b \sim 0.5\,\mu\mathrm{m}$). The squeezing effect is also more pronounced for larger values of Ω, i.e., for Josephson junctions with larger E_J. These points have been demonstrated in Fig. 2 (b), where we have plotted the uncertainty ΔP in the momentum quadrature as a function of the Josephson coupling energy E_J and the ratio b/a of the geometric sizes of the phase qubit. (In the circuit QED systems based on the superconducting circuits,[31] the theoretical limit is known to be $g \sim 1$ as well.) The strong coupling in our scheme is possible because the

vibrational mode directly modulates the largest energy scale (E_J) of the phase qubit. Similar idea has been explored in Refs. [8, 32–35] but using the flexural vibrations of nanomechanical beams. Note that even for the moderate values of Q-factor, $Q \sim 10^3$ [28], the condition for the *conventional* strong coupling limit ($g\sqrt{\Omega\omega} \gg \gamma$) is easily satisfied in our case, $g\sqrt{\Omega\omega}/\gamma \sim 10^3$.

V. NOISE ANALYSIS

It is important to calculate and compare the angle detection limit, for example that of optic interferometer, and the fundamental limits due to thermal and quantum fluctuation. The deflection angle of a torsional resonator can be detected by measuring the displacement at the end of its wing with a fiber-optic interferometer. A low-power($\sim 1\,\mu$W) laser light from a fiber is focused by a lens system to a micron spot on the backside of the resonator, and reflected to couple back into the fiber, giving a displacement-sensitive signal. For the following analysis, we consider a silicon resonator, coupled to a superconducting qubit, consisting of a $2 \times 2\,\mu\mathrm{m}^2$ rectangular paddle suspended by narrow beams on both edges, which are $2\,\mu$m long with a $(100\,\mathrm{nm})^2$ cross-section. To minimize heating effect due to the probing light, it may be necessary to deposit a high-purity silver thin film on the backside, which will play roles of a excellent reflector and a heat sink at a low temperature of 20 mK. From our estimation, 50 nm-thick silver coating may reduce the temperature difference from a cryogenic bath down to 5 mK, while increasing a torsional spring constant by 7 %. Recently, a Fabry-Perot (FP) interferometry with a miniaturized hemi-focal cavity, developed for micrometer-sized cantilevers, was reported to have a remarkably small noise floor as 1 fm $/\sqrt{\mathrm{Hz}}$ at 1 MHz with a decreasing tail at higher frequencies. [36] The effective noise bandwidth of the torsional oscillator of our interest is estimated to be $\Delta\omega = \omega/Q \sim 2\pi \times 2.4\,\mathrm{kHz}$ if $Q \approx 5000$, a typical value for a micrometer-sized oscillator. [29] The angle detection limit for our oscillator is

$$\Delta\theta_{\mathrm{FP}} = 2\sqrt{\frac{S_{\mathrm{FP}}\Delta\omega}{2\pi d^2}} \approx 5 \times 10^{-8}\,\mathrm{rad}, \tag{9}$$

where $d \approx 2\,\mu$m is the lateral size of the qubit and $\sqrt{S_{\mathrm{FP}}} \approx 1\,\mathrm{fm}/\sqrt{\mathrm{Hz}}$ is the noise floor of the FP interferometry. The detection limit is comparable or even smaller than the quantum fluctuation angle $\theta_1 \sim 10^{-7}$ rad (for $I \sim 10^{-28}\,\mathrm{kg{\cdot}m^2}$) or larger, and enough to measure the quantum fluctuations.

The thermal fluctuation of angle originates from the thermal energy stored in mechanical vibration energy of the torsional resonator, and can be estimated as

$$\theta_T = \sqrt{k_B T / I\omega^2}\,. \tag{10}$$

At an experimentally accessible low temperature of 20 mK and with $I \sim 10^{-28}\,\mathrm{kg{\cdot}m^2}$, for instance, the torsional resonator is predicted to vibrate up to $\theta_T \approx 6.2 \times 10^{-7}$ rad. Therefore, the thermal fluctuation would be the main limitation to detecting the quantum vibration. The ratio of the thermal to quantum fluctuation is only ~ 7 at 20 mK, and can be improved even further by lowering cryogenic temperature or by using optical or microwave cooling technique [37]. The quantum temperature $T_Q = \hbar\omega/k_B$ is a border where the torsional resonator enters the quantum regime, and yields 0.37 mK. So, the thermal occupation factor $N = T/T_Q$ is ~ 60 when the torsional resonator is at a temperature of $T = 20$ mK, which can be also found from an experimentally observed fluctuation angle θ_T by $T = I\omega^2\theta_T^2/k_B$.

VI. CONCLUSION

We have proposed a scheme of strong and tunable coupling between a superconducting phase qubit and the nanomechanical torsional resonator. The torsional resonator directly modulates the largest energy scale (the Josephson coupling energy) of the phase qubit, and the coupling strength is achievable. We have analyzed the quantum correlation effects in the torsional resonator as a result of the strong coupling to the phase qubit. We have also provided the noise analysis, which shows that our scheme is feasible experimentally at the level of present technology.

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